Wind and Waterspray

Modeling, Group Assignment

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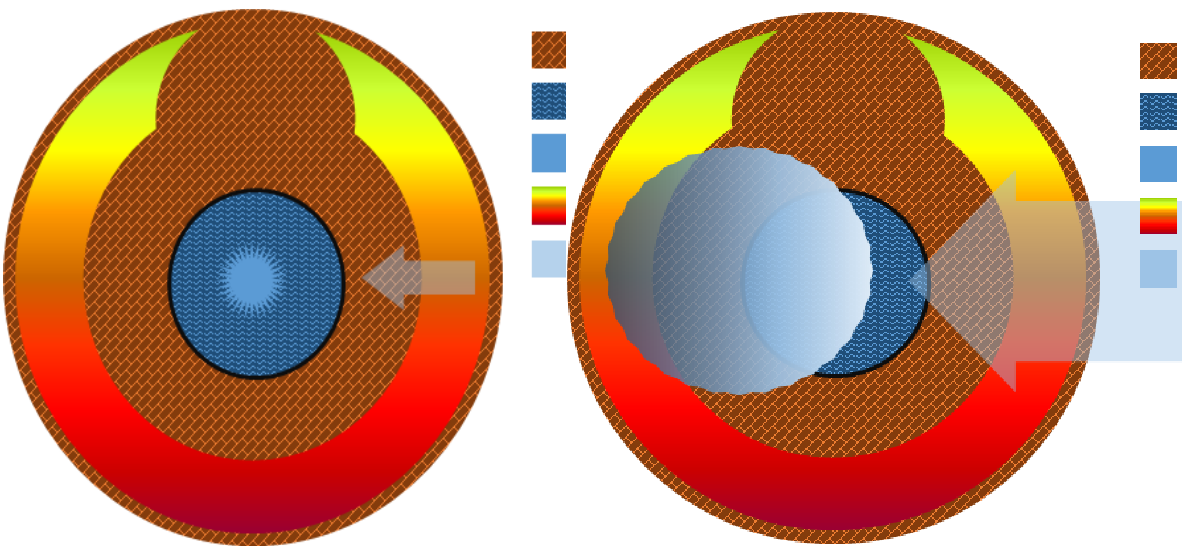
Summary

This report is going to look at a fountain on a round square. The problem with this fountain is that the wind on the square blows the water droplets onto the terraces surrounding the fountain. We had to solve this problem by building a model that calculates what the maximum height of the fountain can be, at certain wind speeds, in order to not wet surrounding people. After analyzing the problem, making assumptions, setting up formulas and getting to the result we were able to find out the maximum water displacement regarding to a certain initial velocity of the water and certain wind speed. By means of discussion and reflection we found out where our model was not 100% complete and we came up with solutions and changes to be made in order to make the model more complete and make it closer to the reality.

Context

On a round open square there is an amazing ornamental fountain which is surrounded by cafeteria. It sprouts water up high in the air and in the evening the fountain is even illuminated, it is a real spectacle. The fountain is a real tourist attraction and it is the main reason why people come to terraces of the cafes and restaurants.
However, on windy days, the water of the fountain gets blown away upon the visitors sitting on the terraces or walking by.
This causes people to leave the square. You could say the wind and the fountain rule the square, where men used to get together.
(see figure 1)



<u>Figure 1</u> *Square with little/ a lot of wind*

Problem Definition and Purpose

If there is a strong wind, the wind blows the water of the fountain onto the people who are sitting on the terraces or are passing by the fountain. In order to prevent this from happening we have to make a model that will help us to determine how high the water should sprout at a certain moment. This height of the fountain is corresponding with a maximum wetting area, which is caused by droplets of water blown away by the wind. We have to find the relationships between the wind, wetting area and height of the fountain. Our goal is to build a model which can predict the preferable height of the water jet of the fountain, based on these found relationships.

Sub questions

1. If we fix the height of the jet, what should be the maximal radius (relative to the middle of the fountain) of the wetting area under a certain wind condition?
2. If the wind conditions stay constant what would be the radius of the wetting area at different heights?
3. Under what conditions do people enjoy the fountain (think of how close do they dare to come and what minimum height makes the fountain a tourist attraction)? In other words, what is the experience at a certain height and wind?
4. How does the wind direction affect the water droplets to fall?

Assumptions

- You can look at the fountain in 3D by looking at where the wind comes from, but with a round square we assume the wind has the same effect whatever the wind direction is. So, now we have made this assumption we can look at the fountain in 2D. The terrace is also round and therefore the displacement is the same in every direction.
- The initial speed of the water from the fountain is constant, because the water pressure is constant so we do not have to take that into account.
- We assume that the wind always comes from one direction at a time.
- The wind only affects the droplets at the top of the fountain.
- The only forces we consider for our model are gravity and the wind force.
- We leave out the air resistance on the water droplets.
- We assume that the optimal height of the fountain is high enough for tourists to enjoy the fountain. Also the experience is not affected by the wind.
- Due to the shape and size of a water droplet it cannot receive full energy from the wind. Therefore the wind speed in x direction is larger than the speed in x direction of the water drops. We assume that the loss of energy is 85 % so that the water has 15% speed of that of the wind.

Conceptual model: Concepts properties, values and relations

Due to the assumptions we can now see the problem as a trajectory from the water of the fountain caused by the velocity of the wind (see figure 2).

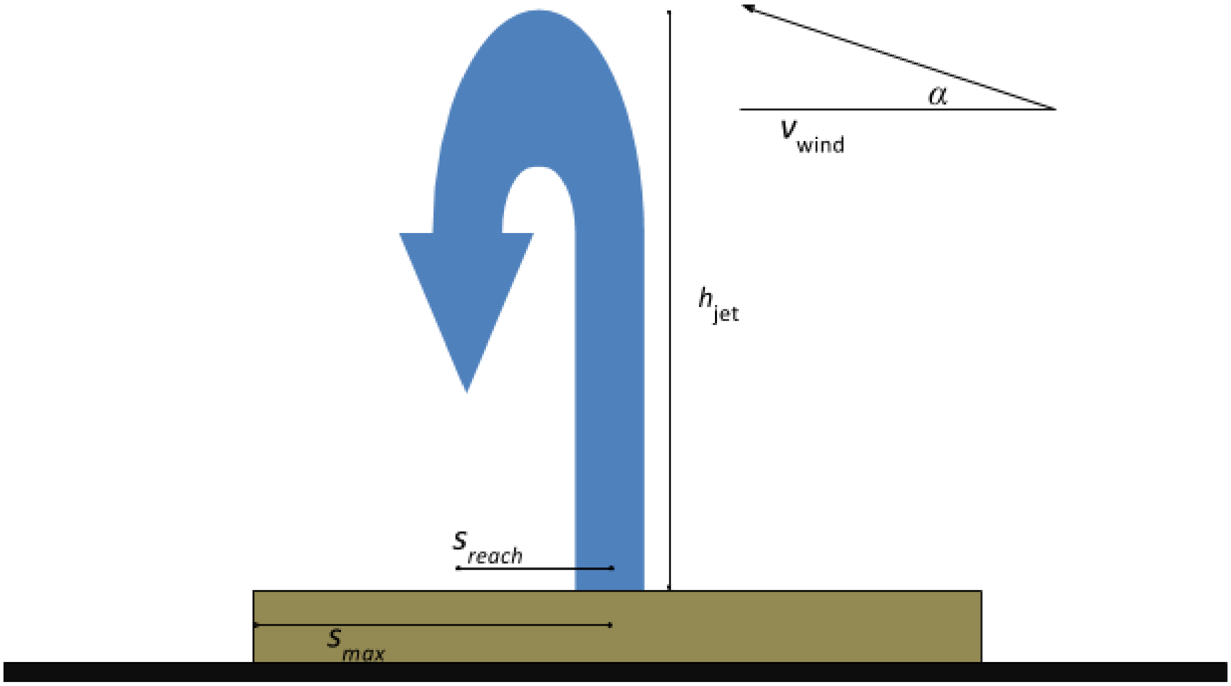


Figure 2 *trajectory 2D fountain*

We named the velocity of the wind V_{wind}. The height of the fountain is called H_{jet}. S_{reach} is the displacement of the water caused by V_{wind}. S_{max} is the maximal displacement that is allowed to not wet the visitors of the square. Also below a list of all the entities and their relations on the next page.

Entities	**Properties**	**Quantities**	**unit**
V_{wind}	Wind speed	Speed	m/s
Alpha	Wind angle	Angle	Degree
H_{jet}	Water height	Height	m
S_{reach}	Waterdisplacement	Displacement	m
t	Time	Seconds	S
g	Gravity	Acceleration	m/s^2
S_{max}	Water reach	Max. Displacement	m

Table: *list of entities*

- As wind is a force of nature, both the windspeed (V_{wind}) and the angle(Alpha) are 2 variables we cannot influence.
- Gravity (g) is a force of nature, but it is constant yet we cannot influence it.
- The initial speed of the water from the fountain, as stated in our assumptions is a constant which we cannot influence.
- The waterheight (H_{jet}) and waterdisplacement (S_{reach}) we can adjust according to the windspeed and our jet.
- Time(t) is a constantly changing thing, where to place this is hard, we can't adjust it what's for sure.
- The water reach (S_{max}) is a constant which we cannot influence because this is related to the size of the fountain.
- The windspeed and the waterheight influence the waterdisplacement, the force of gravity and the windspeed also influences the waterheight, yet in different ways. Gravity does this in a direct way but the windspeed in an indirect way.

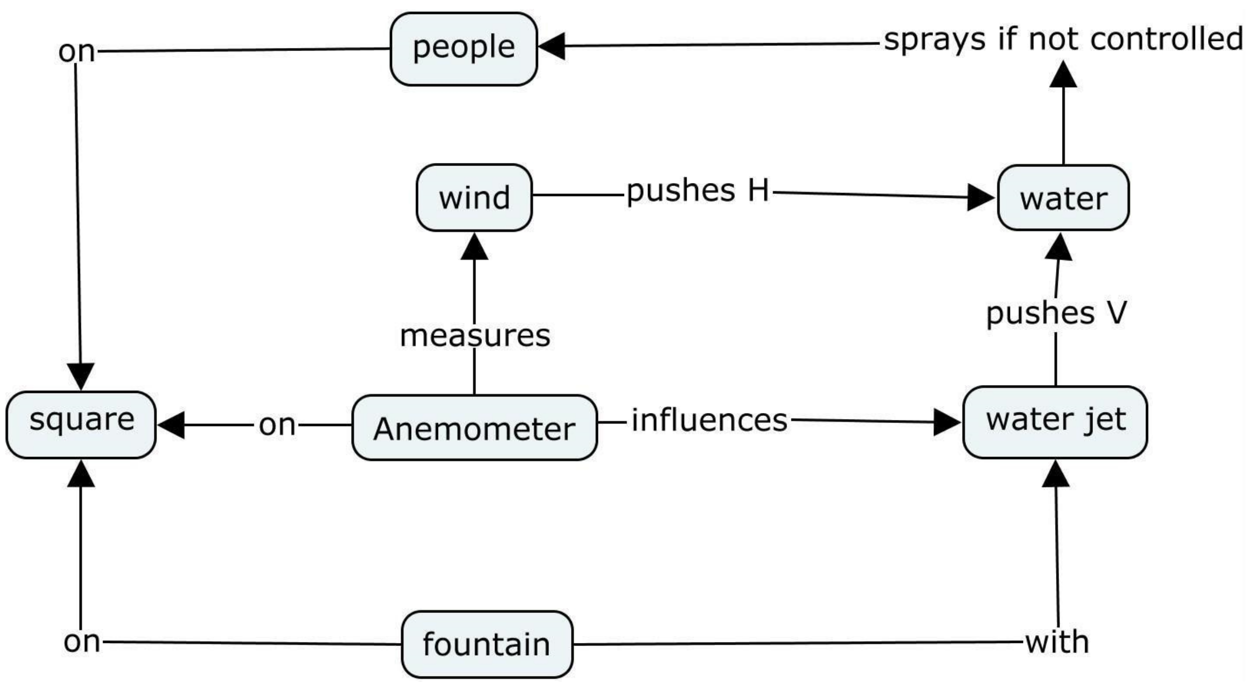
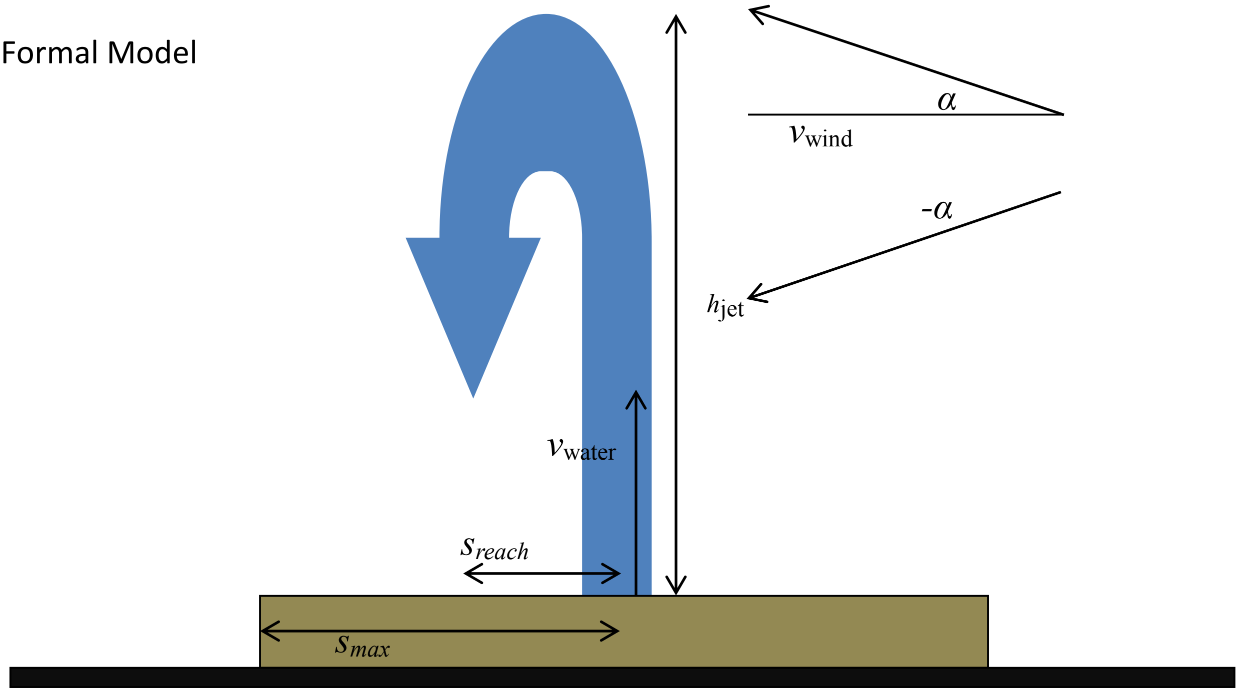


Diagram *Relations of entities*

Formal Model



s_{max} = the radius of the pond and the maximum permissible value of s_{reach}.
h_{jet} = the height of the water jet
v_{wind} = the windspeed
v_{water_0} = the initial speed of the water jet
α = the angle that the wind makes relative to the Earth's surface.

Formulas:

The standard formula of distance is $s = \frac{1}{2} \cdot a \cdot t^2$. Because we assume that the air resistance is negligible and we're going to use this formula in the y direction, we can say that $a = g$ (the gravitational force) and s is the height of a water drop at a certain t. So$h_t = \frac{1}{2} \cdot g \cdot t^2$. However, the water drop starts at a certain height and falls down. That's way we have to add the height of the water jet to our formula like: $h_t = h_{jet} - \frac{1}{2} \cdot g \cdot t^2$. There is a minus sign, because the vector of the water drop is pointing downwards. The last thing we have to add is about the direction of the wind. The wind can give a little upward direction first to the water drop. We can compromise that by formulating the formula like:$h_t = h_{jet} + v_{wind} \cdot \sin\alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2$. Alpha is the angle that the wind makes relative to the Earth's surface

So in the end we have this formula:

$$h_t = h_{jet} + v_y \cdot t - \frac{1}{2} \cdot g \cdot t^2$$
$$v_y = v_{wind} \cdot \sin\alpha$$

h_t = the height of a water drop at a certain time.
t = the time it takes for a water drop to get from the top of the jet back to the surface.
We determine that $h_{t_{max}} = 0$. (because the surface of the pond is 0 meters high)
v_y = the velocity of a water drop at a certain t in the y direction.
α = the angle that the wind makes relative to the Earth's surface.

The second formula is used to describe the movement of a water drop in the x direction. The standard formula for this is $s = v \cdot t$. In this module we write s as s_{reach} and v as v_{wind}. If we pay attention to the wind direction we just at a cosines; $s_{reach} = v_{wind} \cdot \cos\alpha \cdot t$. It is possible that the droplets are taken up.

Calculation 1:

We calculate how long it will take for a water drop to get back from the top of the water jet to the surface of the pond.

$$h_t = h_{jet} + v_y \cdot t_{max} - \frac{1}{2} \cdot g \cdot {t_{max}}^2$$
$$h_t = 0$$
$$v_y = v_{wind} \cdot \sin\alpha$$

So $$h_{jet} + v_{wind} \cdot \sin\alpha \cdot t_{max} - \frac{1}{2} \cdot g \cdot {t_{max}}^2 = 0$$
$$-\frac{1}{2} \cdot g \cdot {t_{max}}^2 + v_{wind} \cdot \sin\alpha \cdot t_{max} + h_{jet} = 0$$

We can use the abc-formula to continue:

$$x_{1,2} = \frac{-b \pm \sqrt{\mathrm{D}}}{2 \cdot a}$$

In this:

$$\mathrm{D} = (b)^2 - 4 \cdot a \cdot c$$
$$x = t_{max}$$
$$a = -\frac{1}{2} \cdot g$$
$$b = v_{wind} \cdot \sin\alpha$$
$$c = h_{jet}$$

$$\mathrm{D} = (v_{wind} \cdot \sin\alpha)^2 - 4 \cdot -\frac{1}{2} \cdot g \cdot h_{jet} = (v_{wind} \cdot \sin\alpha)^2 + 2 \cdot g \cdot h_{jet}$$

$$t_{max} = \frac{-v_{wind} \cdot \sin\alpha \pm \sqrt{(v_{wind} \cdot \sin\alpha)^2 + 2 \cdot g \cdot h_{jet}}}{-g}$$

= the time it takes for a water drop to get from the top of the jet back to the surface.

Calculation 2:

We use the t_{max} to create the formula that can describes where the water drop lands in the pond or outside the pond.

$$s_{reach} = v_{wind} \cdot \cos\alpha \cdot t$$

$$t_{max} = \frac{-v_{wind} \cdot \sin\alpha - \sqrt{(v_{wind} \cdot \sin\alpha)^2 + 2 \cdot g \cdot h_{jet}}}{-g}$$

So $$s_{reach} = v_{wind} \cdot \cos\alpha \cdot \frac{v_{wind} \cdot \sin\alpha + \sqrt{(v_{wind} \cdot \sin\alpha)^2 + 2 \cdot g \cdot h_{jet}}}{g}$$

Because the water droplets can't get the same speed of the wind, it is necessary to add a constant; we called it β. It is a percentage of the wind speed.

$$s_{reach} = \beta \cdot v_{wind} \cdot \cos\alpha \cdot \frac{\beta \cdot v_{wind} \cdot \sin\alpha + \sqrt{(\beta \cdot v_{wind} \cdot \sin\alpha)^2 + 2 \cdot g \cdot h_{jet}}}{g}$$

If $s_{reach} > s_{max}$ it is necessary to change h_{jet}.

We used a previous formula (located below this paragraph) where alpha was not included to get rid of the $\pm$-sign. We took alpha as zero an calculated s_{reach} in both formulas and came to the conclusion that there should be a minus sign.

$$s_{reach} = \sqrt{\frac{{v_{wind}}^2 \cdot h_{jet}}{\frac{1}{2} \cdot g}}$$

h_{jet} is an output of the initial speed of the water jet. If you want to change the height of the water jet, you have to change the initial speed.

This can be explained in this formula: $s = \frac{1}{2} \cdot a \cdot t^2$. We can call $s = h_{jet}$ and $a = g$.
So$h_{jet} = \frac{1}{2} \cdot g \cdot t^2$. In the formula t is the time it takes for a water droplet to get to the top of the jet. We can determine that $t = \frac{v_{water_0}}{g}$ and we will get $h_{jet} = \frac{1}{2} \cdot g \cdot (\frac{v_{water_0}}{g})^2$.

$$h_{jet} = \frac{1}{2} \cdot g \cdot \frac{{v_{water_0}}^2}{g^2}$$

$$h_{jet} = \frac{g \cdot {v_{water_0}}^2}{2 \cdot g^2}$$

$$h_{jet} = \frac{{v_{water_0}}^2}{2 \cdot g}$$

Problem Statement

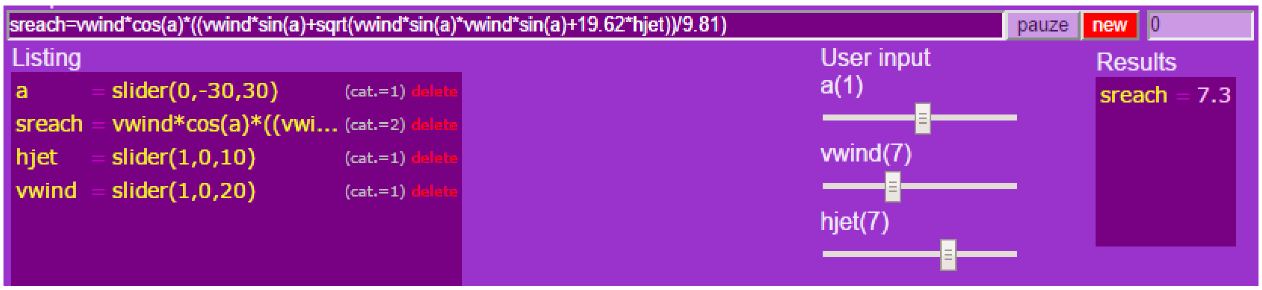
Now we are able to give the problem statement in formal terms:
There is a fountain on a round square with a waterjet, which reaches H_{jet}. The wind produces V_{wind} which blows water droplets of H_{jet} onto people surrounding the fountain. In order to prevent this from happening we have to make a model that will help us to determine how high H_{jet} should preferably be at a certain V_{wind} .

H_{jet} is corresponding with a wetting area that has a displacement of S_{reach}, which is caused by V_{wind}. In order not to wet the surrounding people the displacement has a maximum S_{max}. If S_{reach} gets equal to S_{max} , H_{jet} has to be lowered. So the goal of our model is to predict the preferable H_{jet} at a certain V_{wind}.

Simulation and Results Graph

With Accel we made a small simulation of values, where the wind can have an maximum angle of 30 and -30 degrees, the height of the water h_{jet} can be 10 meters and the wind speed is maximum for 20 m/s.

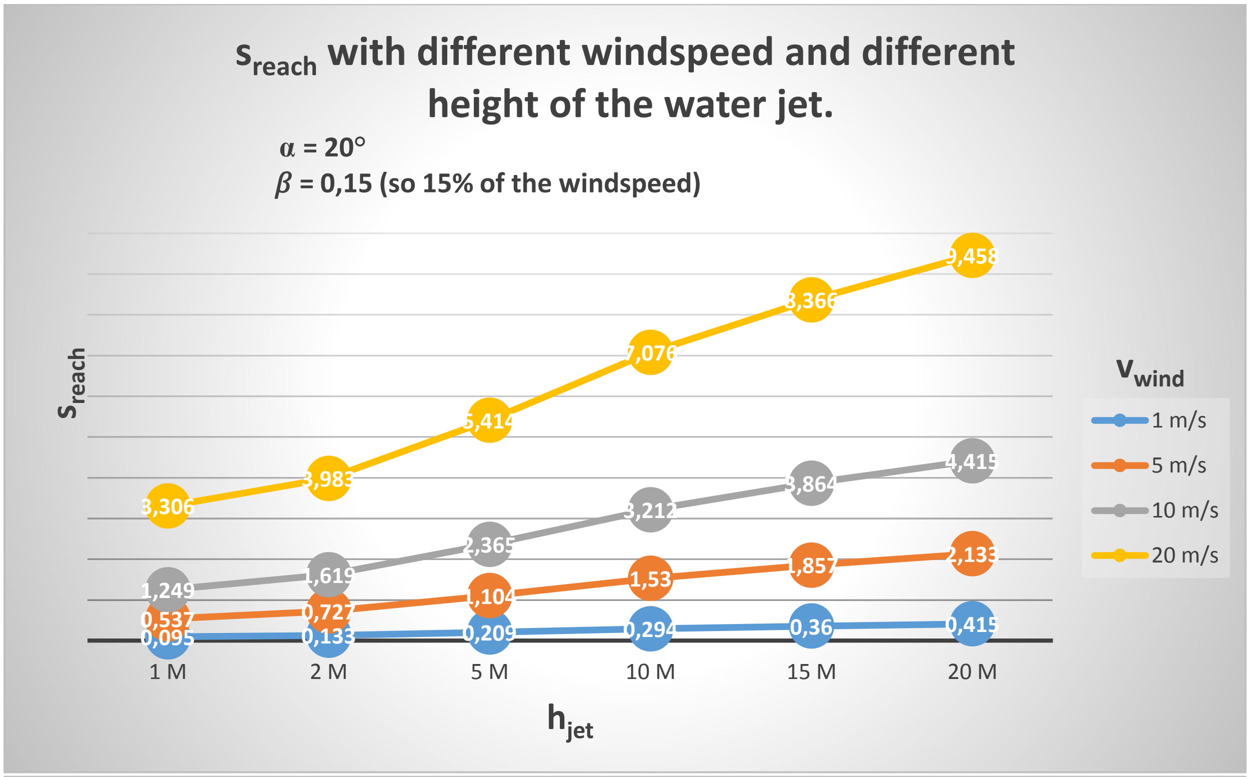
We used the following formula:
sreach=vwind*cos(a)*((vwind*sin(a)+sqrt(vwind*sin(a)*vwind*sin(a)+19.62*hjet))/9.81)

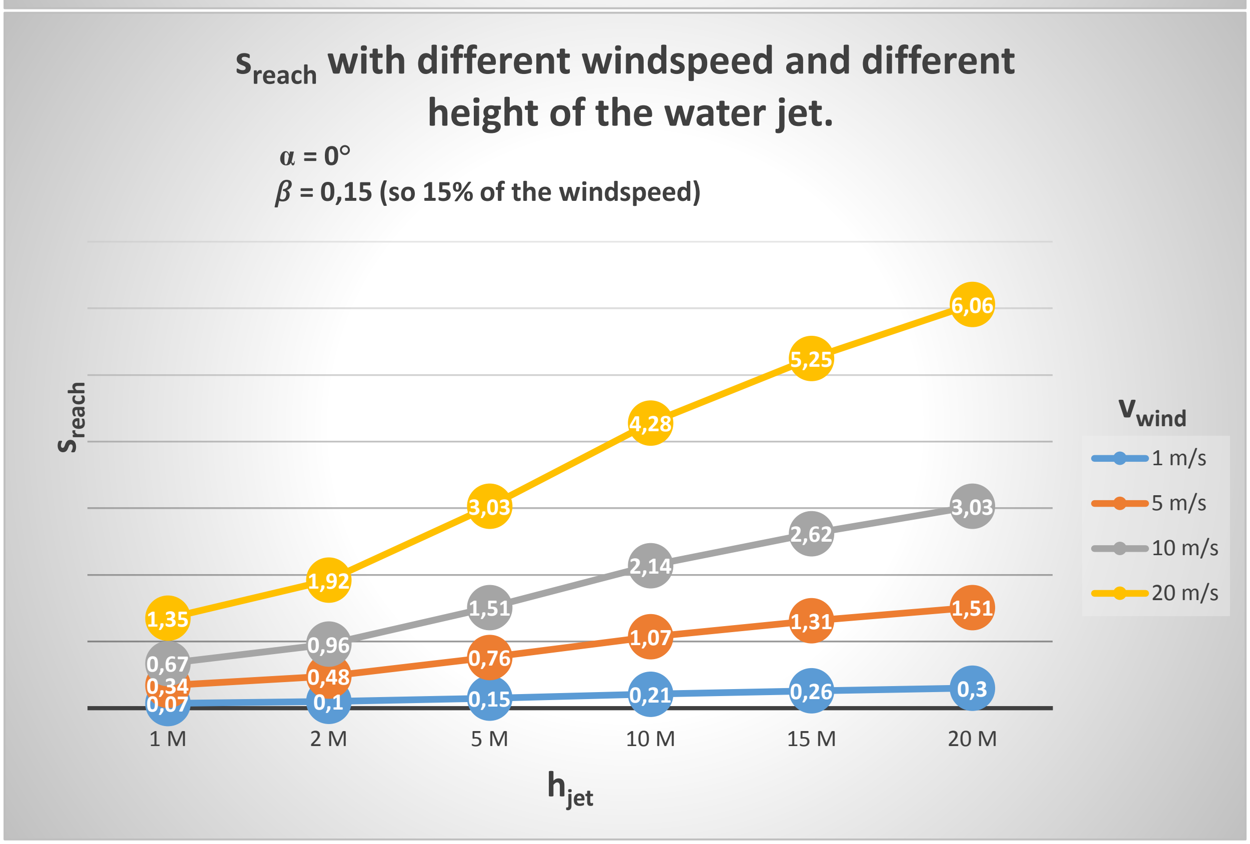


After this first simulation we got some non-realistic results, so we figured there was still a problem to be looked at. We reckoned that the water droplets do not reveive the same speed at the top as the wind. So we made an assumption that the water droplets receive 15% of the speed of the wind.

We came up to the following formula:
sreach=b*vwind*cos(a)*(((b*vwind)*sin(a)+sqrt((b*vwind)*sin(a)*(b*vwind)*sin(a)+2*9.81*hjet))/9.81)

Of these results we made two graphs with α = 0° and α 20°:





<u>Graph 1</u> *Calculations with α = 0 and α = 20*

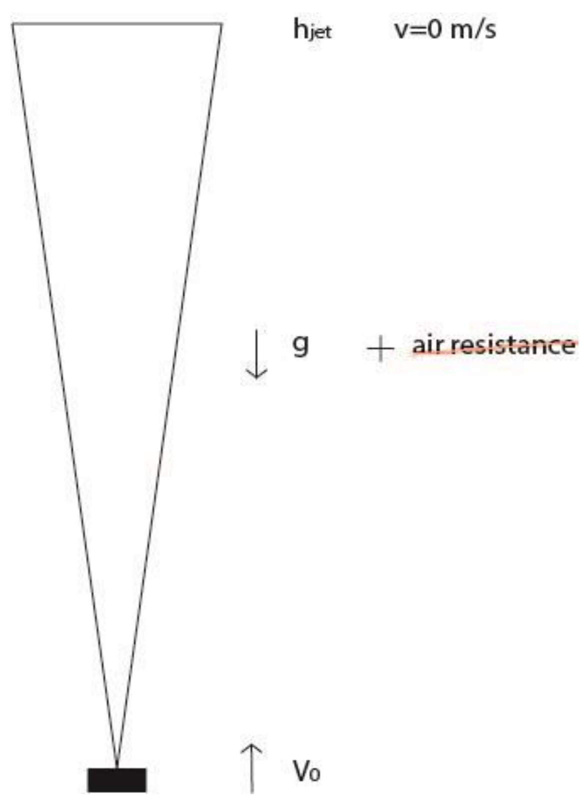
As you can see in the graphs above alpha has a big influence on sreach. With an alpha of 20 degrees sreach gets just about 1.5 times bigger.

Discussion after Conceptual Model

In the conceptual model we left out a few influences to simplify the problem case:
We made the problem that could have been a 3-D model into a 2-D model by assuming and stating that the fountain was round. This causes the wind to be able to blow away droplets in a measurable circle and therefore the displacement easier approximated in every dimension. This is a realistic assumption since it is possible to make a round pond.

We also chose to not take the experience people have with the fountain in account, because this is a very subjective matter and therefore not easily testable and verifiable. However we do expect that if we optimise the height of the fountain that this will give the most spectacular experience. So the goal of finding the best way to make the fountain enjoyable is achieved by our model.

As you can see in the image, the volume at the top of the fountain is bigger than at the bottom. The reason for this is that the initial speed of the fountain is higher than the speed at the top (which is zero). The bigger volume of the fountain will affect the displacement of the water droplets. We did not take this into account.



The estimated air resistance does not have enough effect on the water droplets to make a notable difference. Therefore it is realistic that we did not take it into account. We did not take this into account because the water droplets are so small and light the air resistance does barely have any influence of it.

Discussion after Formal model

- We did only use half of the trajectory, from top to bottom, instead of using a complete trajectory and corresponding formula.
- First we didn't take in account the wind direction, however to make our formula more complete we integrated the variable of wind direction.
- After our first results we saw that they were pretty large, so we created an constant Beta, to create a transmission from the wind force into the force onto the water droplets.
- We could have integrated the trajectory and air resistance in our formula.

Discussion after the Result

After the model calculations -which were fine, from a numerical point of view- we saw that our results were not valid/realistic: the displacements of the water droplets were too big. So we looked at the model what could be done to make the results more realistic. We integrated, as said earlier in the discussion, a constant (Beta) for the transmission of wind force onto the water droplets.

Discussion after the solution of the initial problem

When we look back at the initial problem where we want to predict the preferable height of the water jet of the fountain depending on the wind speed, we see that we solved the problem with our model. We are able to calculate the displacement of the water depending on the wind speed. We should mention that our solution is only valid when the pond of the fountain is round.

Reflection on our assignment

Extension

We know that our model is not 100% complete yet. In order to make our model more complete we could make a few extensions by focussing on things we left out of account. Things we let out of account:

- **air resistance.** We let this out of account because it wouldn't influence the model at an extent that would influence the results. But although it doesn't influence the model, it would have been more complete if we took it into account.
- **experience of people passing by.** Everyone could experience the fountain different. When is something still interesting to watch for different type of persons?
- **bigger volume.** As said in the discussion, we think that the volume of all the water together at the top is bigger than at the bottom due to an change in the initial speed.

These are small things to take in account, but more importantly are the extension we would have tried to implement if we had more time to work on the model:

- **work with the whole trajectory.** We thought of using the whole trajectory, so let the wind influence the water jet from the bottom and not from the top of the waterjet.
- **weather conditions.** Something we didn't pay attention to at all, is the influence of the weather circumstances. We are aware of the fact that for example rain, temperature and air pressure could influence the height and the water displacement of the fountain.

What aspects of your work are you proud of?

We are proud of our created formula which involves several variables like the wind direction.
Also we are proud of that what we have, we understand. We have created a balance of what was achievable and understandable for us within the given time. Also our assumptions which we created fairly early on seemed to cover problems which we encountered later on. This means our assumptions were good and useful.
Our conclusions during the report (e.g. that the s_{reach} doubled no matter the height of h_{jet}) we are proud of, looking at the outcomes and actually think about it has given us some more depth in our report.
Also, the fact that we are with 4 people instead of 6, we are very happy with what we achieved with our final report.

What have we learned?

We have learned that a good model has to have a lot of assumptions. These assumptions need to deal with every aspect of the situation. There were always some new perspectives which you had to take in account. You need these assumptions to keep the report understandable for everyone. Another important aspect is that we have learned how to create a formula out of scratch. With the help of our knowledge from middle school and our new gained knowledge for this report we are able to create a suitable formula to describe everyday situations.
Working on this report forced us to work with the terms we learned in the lectures. For us this worked good because you see the terms moreover and don't learn only the terms in sentences and by lectures. We really had to dive in the meaning of terms in order to work with them in the assignment. In hindsight, we think that having this assignment worked out better than only having to learn by lectures.

List of definitions

Alpha: wind angle
Beta: constant for the transmission of windforce on the water droplets
G: Gravity
Hjet: Water height
Smax: maximum water displacement
Sreach: Water displacement
Vwind: Wind speed